### **Conclusions**

A general solution has been obtained for the steady subcritical potential flow past a slender ship in shallow water with depth variations of the order of the slenderness parameter. The contributions to the pressure distribution, vertical force and trim moment from the depth variation are of the same magnitude as the constant-depth results of Tuck.<sup>2</sup> The pressure has been obtained to  $0(\epsilon)$  and the force and moment to  $0(\epsilon^2)$ . For the special case of depth variation only in the streamwise direction, the pressure is seen to be proportional to the local depth deviation from the mean position of the bottom.

#### References

<sup>1</sup>Plotkin, A., "Flow Past an Anchored Slender Ship in Variable-Depth Shallow Water," *Journal of Hydronautics*, Vol. 9, 1975, pp. 103-106.

<sup>2</sup>Tuck, E. O., "Shallow-Water Flows Past Slender Bodies," *Journal of Fluid Mechanics*, Vol. 26, 1966, pp. 81-95.

<sup>3</sup> Wehausen, J.V. and Laitone, E.V., "Surface Waves," *Handbuck* 

Wehausen, J.V. and Laitone, E.V., "Surface Waves," *Handbuck der Physik*, edited by S. Flugge, Vol. 9, Springer-Verlag, Berlin, 1960, pp. 446-778.

pp. 446-778.

<sup>4</sup> Van Dyke, M., Perturbation Methods in Fluid Mechanics, Academic Press, New York, 1964.

# **Hydroelastic Ichthyoid Propulsion**

Michael P. Paidoussis\*

McGill University, Montreal, Quebec, Canada

# Introduction

THE use of undulating plates as a means of aquatic propulsion has been known for a long time. However, the development of this and of other propulsive methods has been discouraged by the outstanding simplicity and efficiency of the propeller. Nevertheless, in some cases, propulsion by undulating plates may have distinct advantages over the propeller, namely; a) in deep water applications, where sealing of rotating propeller shafts is difficult, and b) in muddy and weed-infested environments.

In a recent study, Botman<sup>1</sup> investigated several aspects of the feasibility of propulsion by undulating plates. In his experiments, Botman utilized a submerged cantilevered plate to drive a catamaran, the undulation of the plate being generated by a motor through mechanical links. He concluded that this is a feasible, but low efficiency, propulsive method which may find specialized application.

This Note presents an alternative means of generating the undulatory motion. This is by fluid flow through pipes attached on the plate, thus eliminating the mechanical links, and affording better control over the waveform of the undulating plate. This has been named "hydroelastic ichthyoid propulsion", as it is based on harnessing the energy associated with a hydroelastic instability, and the resulting motion of the plate is similar in form to that of swimming fish (ichthys).

# Mechanism of Hydroelastic Ichthyoid Propulsion

It is well known that a cantilevered pipe conveying fluid becomes unstable by flexural flutter at sufficiently high flow velocities.<sup>2,3</sup> Herrmann and Nemat-Nasser<sup>4</sup> studied some aspects of the hydroelastic behavior of such systems by con-

sidering the dynamics of thin plates on which flexible pipes conveying fluid were attached. This system exhibits the same stability characteristics as a pipe by itself (although torsional instabilities may also occur for some configurations). As discussed by Paidoussis, <sup>5</sup> energy transfer from the fluid to the pipe or vice-versa occurs accordingly as the flow velocity is above or below the critical flow corresponding to the onset of flutter. At supercritical flow velocities energy is transferred from the flowing fluid to the pipe. This results in amplifying small free motions of the system. The ensuing limit-cycle motion is essentially in the second mode of the system, but with a nonstationary waveform involving a downstream propagating wave with increasing amplitude. This resembles the anguilliform mode of swimming of slender marine animals. <sup>6</sup>

It occurred to the author that a thin plate with attached flexible pipes, or a moulded composite structure, conveying fluid supplied by a pump, and oscillating in a (supercritical) limit cycle could well be an alternative means of propulsion to that of a mechanically driven plate. As has been shown by Lighthill, the feasibility of anguilliform swimming of marine animals depends vitally on a downstream propagating wave. This in fact should have a phase velocity higher than the forward swimming speed. Therein lies the main advantage of the proposed propulsion mechanism: a downstream propagating wave is an inherent characteristic of the undulatory motion of the plate.

# **Model Experiments**

A limited experimental program was undertaken to test the feasibility of this method of propulsion. A simple catamaran structure was built, with a  $\frac{1}{2}$  hp motor-pump unit supported between the floats, and powered by an overhead electric line, as shown in Fig. 1. The plate was typically 6-15 cm wide, 25-60 cm long, and with thickness  $t \sim 0.25$  mm. Two tygon pipes (0.635 cm diam, 1.59 mm wall thickness) were clipped symmetrically on either side of the plate, as shown in Fig. 2, and the flow was divided equally between them by a specially made adaptor. The experiments were conducted in a tank of 91.5 cm  $\times$  91.5 cm (3 ft  $\times$  3 ft) cross-section and 15 m long. Spoilers at the rear of the catamaran controlled the forward speed of the craft.

Propulsion in such an arrangement occurs even without undulation of the plate, simply by the change in momentum of the internal flow. Thus the experiments were confined to deciding whether propulsion with undulatory motion could be more efficient than without. Clearly, if this were shown not to be the case, hydroelastic ichthyoid propulsion would be proven a failure. To this end, for each set of experimental parameters, the forward speed was measured with the plate undulating and not undulating, the latter being achieved by adding a stiffener to the upper edge of the plate (shaped so that the increase in drag was negligible).

The experimental procedure was as follows: the catamaran was held, and the power turned on at a specific setting of flow through the pipes. The catamaran was then released, and it moved forward. Allowing about 4 m for a constant speed condition to be reached, the motion was timed over the next 8.5 m, establishing a value of U. Approximate measurements of the phase velocity and frequency could be made by direct observation of the plate over a certain distance.

# **Observations and Results**

There was no difficulty in achieving propulsion, typically with a forward speed  $U \sim 1$  m/sec. The undulation of the plate was as expected, and as observed in previously conducted experiments in a water tunnel, a backward propagating wave was clearly present with downstream-increasing amplitude. Typically, the undulation frequency  $\omega$  was 15 rad/sec and the wavelength was  $0.6\ell$ ,  $\ell$  being the length of the plate, so that the reduced frequency,  $\sigma = \omega \ell \, 2U \sim 5$ .

Received Nov. 3, 1975. The support of the Defense Research Board of Canada is gratefully acknowledged.

Index categories: Hydrodynamics; Marine Propulsion; Aeroelasticity and Hydroelasticity.

<sup>\*</sup>Associate Professor, Department of Mechanical Engineering.

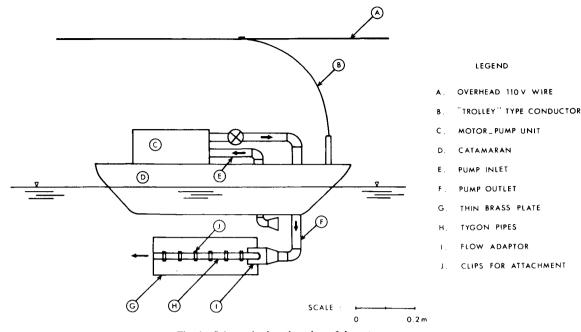


Fig. 1 Schematic elevation view of the catamaran

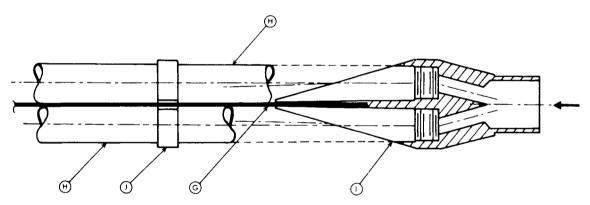


Fig. 2 Top section of the plate, pipes, and adaptor also serving as plate support.

For a constant internal flow, if U was allowed to increase by lowering the drag, the phase velocity became smaller than U, so that undulation impeded rather than aided propulsion. Moreover, if U increased sufficiently, the damping effect of external flow stabilized the plate, effectively neutralizing the destabilizing effect of the internal flow. This could be counteracted by increasing the internal flow velocity, which reestablished undulatory motion. However, if the phase velocity of the undulating plate was higher than U, it was found that undulation could in fact aid propulsion.

Several series of experiments were conducted: varying the internal flow, plate width and length, drag of the craft, etc. The parameter space is clearly quite complex, and no attempt was made to match the drag with the pump characteristics and thrust achieved. Therefore, there was no guarantee that an efficient system would be obtained in these preliminary experiments. Neverless, by comparing the values of U without undulation to those with undulation, it was evident that undulation with the correct characteristics results in higher U. The most effective size of plate was found to be 5 cm wide and 50 cm long, with pipes 43 cm long, yielding 30% higher speeds than with no undulation. This was found to correspond to approximately 60% higher thrust.

# Conclusion

There seems to be little doubt that propulsion by plate undulation generated by internal flow is a feasible proposition,

and that it can be more efficient than propulsion by fluid jet alone. Clearly, the percentage increase in thrust could be greatly improved upon by optimizing the system. Thus, in order to generate an optimum undulatory motion, one could change the shape of the plate as well as vary its stiffness along the length. One could also introduce nonuniform internal flow conduits to replace the uniform pipes, and improve upon the design of the pipe discharge. These, however, will be undertaken in the second phase of the experimental program. It seems clear that, although unlikely to rival the propeller in efficiency, hydroelastic ichthyoid propulsion could find specialized applications for low or medium velocity craft in muddy, weed-infested environments, or at great depths.

## References

<sup>1</sup>Botman, M., "Propulsion by Undulating Plates," *Journal of Aircraft*, Vol. 2, Sept.-Oct. 1965, pp. 456-462.

<sup>2</sup>Gregory, R.W. and Paidoussis, M.P., Unstable Oscillation of Tubular Cantilevers Conveying Fluid. I. Theory," *Proceedings of the Royal Society (London)*, Vol. 293, 1966, pp. 512-527.

Royal Society (London), Vol. 293, 1966, pp. 512-527.

Gregory, R.W., and Paidoussis, M.P. "Unstable Oscillation of Tubular Cantilevers Conveying Fluid. II. Experiments," Proceedings of the Royal Society (London), Vol. 293, 1966, pp. 528-542.

<sup>4</sup>Herrmann, G. and Nemat-Nasser, S., "Instability Modes of Cantilevered Bars Induced by Fluid Flow through Attached Pipes," *International Journal of Solids and Structures*, Vol. 3, 1967, pp. 39-52.

<sup>5</sup>Paidoussis, M.P., "Dynamics of Tubular Cantilevers Conveying Fluid," *The Journal of Mechanical Engineering Science*, Vol. 12, No. 2, 1970, pp. 85-103.

<sup>6</sup>Lighthill, M.J., "Hydromechanics of Aquatic Animal Propulsion," *Annual Review of Fluid Mechanics*, Vol. 1, 1969, pp. 413-446.

<sup>7</sup>Paidoussis, M.P., "Dynamics of Flexible Slender Cylinders in Axial Flow. Part 1. Theory," *Journal of Fluid Mechanics*, Vol. 26, Part 4, 1966, pp. 717-736.

# **Technical Comments**

# Comment on "Turning Moment on a Rotating Disk"

Paul S. Granville\*
Naval Ship R&D Center, Bethesda, Md.

**F**OR the combined presence of laminar and turbulent flow in the boundary layer on a rotating disk, the authors have overlooked a general formula for the moment coefficient. In the notation of the authors this formula published as Eq. (9) of Ref. 2 is:

$$C_M = (C_M)_{\text{turb}} - (R_c/R)^{2.5} [(C_m)_{\text{turb}} - (C_M)_{\text{lam}}]$$

#### References

<sup>1</sup>Shanebrook, J. R. and McCullan, D., "Turning Moment on a Rotating Disk," *Journal of Hydronautics*, Vol. 9, Jan. 1975, pp. 46-47.

Received April 28, 1975.

Index category: Hydrodynamics.

<sup>2</sup>Granville, P. S., "The Torque and Turbulent Boundary Layer for Rotating Disks with Smooth and Rough Surfaces, and in Drag-Reducing Polymer Solutions," *Journal of Ship Research*, Vol. 17. Dec. 1973, pp. 181-195.

# Reply by Authors to P.S. Granville

J. Richard Shanebrook\* Union College, Schenectady, N.Y.

and

David McMullan†
General Electric Company, Schenectady, N.Y.

THE authors thank Dr. Granville for bringing Eq. (97) of Ref. 1 to our attention. We note in closing that Ref. 2 provides comparisons with experimental data for Reynolds numbers in the intermediate range where Eq. (97) of Ref. 1 is of interest.

#### References

<sup>1</sup> Granville, P. S., "The Torque and Turbulent Boundary Layer for Rotating Disks with Smooth and Rough Surfaces, and in Drag-Reducing Polymer Solutions," *Journal of Ship Research*, Vol. 17, Dec. 1973, pp. 181-195.

<sup>2</sup>Shanebrook, J. R. and McMullan, D., "Turning Moment on a Rotating Disk," *Journal of Hydronautics*, Vol. 9, Jan. 1975, pp. 46-47.

Received June 11, 1975.

Index category: Hydrodynamics.

\*Associate Professor, Department of Mechanical Engineering. Member AIAA.

†Engineer, Thermal Design and Performance Analysis, S8G Project, Knolls Atomic Power Laboratory.

<sup>\*</sup>Research hydrodynamicist.